

**B. Math II & III Year**  
**2005 Mid-term Examination – Topics in Optimization**  
**Time 2 hours**

**Date: March 10, 2005**

**Maximum marks: 50**

**Q1 [5 Marks]** Attempt any one of the following two. Identify pure strategy equilibria for the non-cooperative game and in case there is more than one then briefly discuss which one is likely to dominate

a)	<table border="1" style="border-collapse: collapse; text-align: center;"> <tr> <td></td> <td><b>C</b></td> <td><b>D</b></td> </tr> <tr> <td><b>A</b></td> <td>(4,-300)</td> <td>(10,6)</td> </tr> <tr> <td><b>B</b></td> <td>(8,8)</td> <td>(5,4)</td> </tr> </table>		<b>C</b>	<b>D</b>	<b>A</b>	(4,-300)	(10,6)	<b>B</b>	(8,8)	(5,4)
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**Q2 [20 Marks]** Let's go back to the Indian general elections of 2003-4 – specifically to Sonia Gandhi's (referred to as S) decision to refuse prime ministership and BJP's (referred to as B) decision to make a big deal about her foreign origin before she takes her decision. In this political game the payoff comprises of the popularity of the party or a stable government or a combination of the two. For the sake of simplicity let's represent this payoff using an ordinal number from 0 to 10. Here are the rules of the game:

- If B decides to raise the issue just before S declines then S will get the public sympathy (as the public would be angry at B's pettiness) and a payoff of 10 and B will get 0.
- If B decides not to raise the issue just before S declines then B will get a payoff of 5. S will still do well (but now the sympathy wave is not as strong) with payoff of 8
- If S decides to be the PM her payoffs are good (5) but not as good as before. Now if B raises the issue before S accepts the PM position, it is likely to result in making the political coalition members worried about the stability of the government and the payoff for B is 5.
- If B decides not to raise the issue (before S accepts the position) then it enhances party's popular standing but it loses the chance to make the upcoming government unstable. So the net payoffs are: 3 to B and 5 to S.

Represent the game in extensive form, identify the pure strategies for the players, represent the game in a matrix form. State what type of a game this is and then solve it.

**Q3 [10 Marks]** Prove the following theorem: A finite n-person non-cooperative game  $T$  in extensive form which has perfect information possesses an equilibrium point in pure strategies

**Q4 [15 Marks]** Find the characteristic function of the 3-person game with the normal form

		<b>Player 3</b>			
		<b>A</b>		<b>B</b>	
		<b>Player 2</b>		<b>Player 2</b>	
		<b>A</b>	<b>B</b>	<b>A</b>	<b>B</b>
<b>Player 1</b>	<b>A</b>	(1,1,0)	(4,-2,2)	(-3,1,2)	(0,1,1)
	<b>B</b>	(1,2,-1)	(3,1,-1)	(2,0,-1)	(2,1,-1)
	<b>C</b>	(-1,0,1)	(-2,1,-1)	(0,-1,3)	(-3,2,1)